$$\beta = \frac{(N_2 - N_1)c^2}{8\pi\nu^2} \qquad I = I_0 \cdot e^{(\beta-\alpha)2l}$$

04.12.2012

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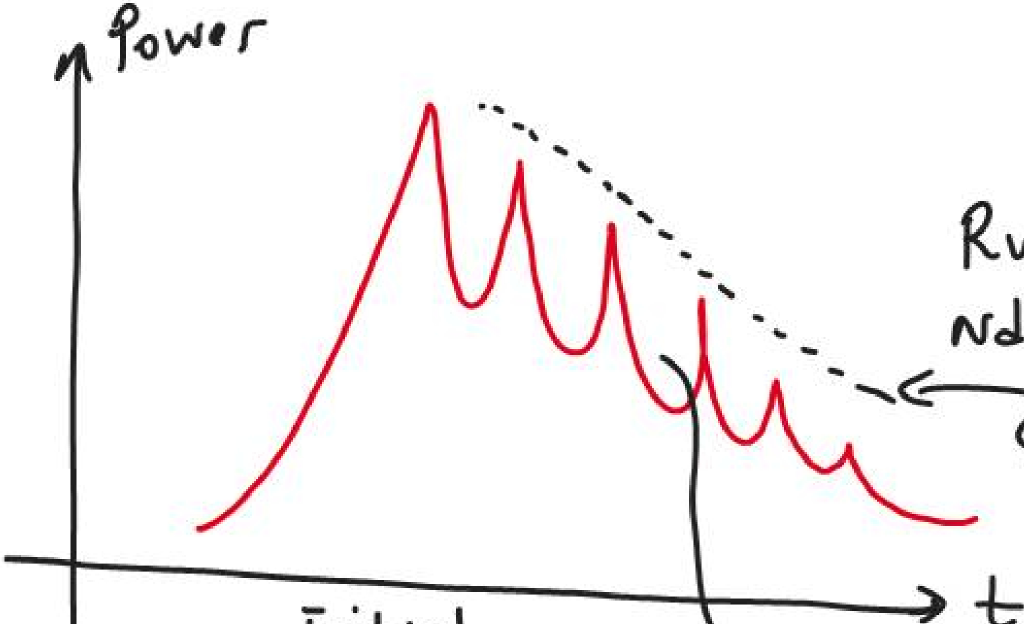
Celal Zaim Çil

PULSED OPERATION

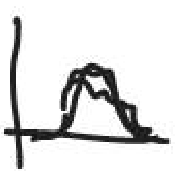


$Cr^{+3}: Al_2O_3$

Ruby
Nd: Glass } Lasers

Gradual decline is due to the diminishing power output of flash lamps

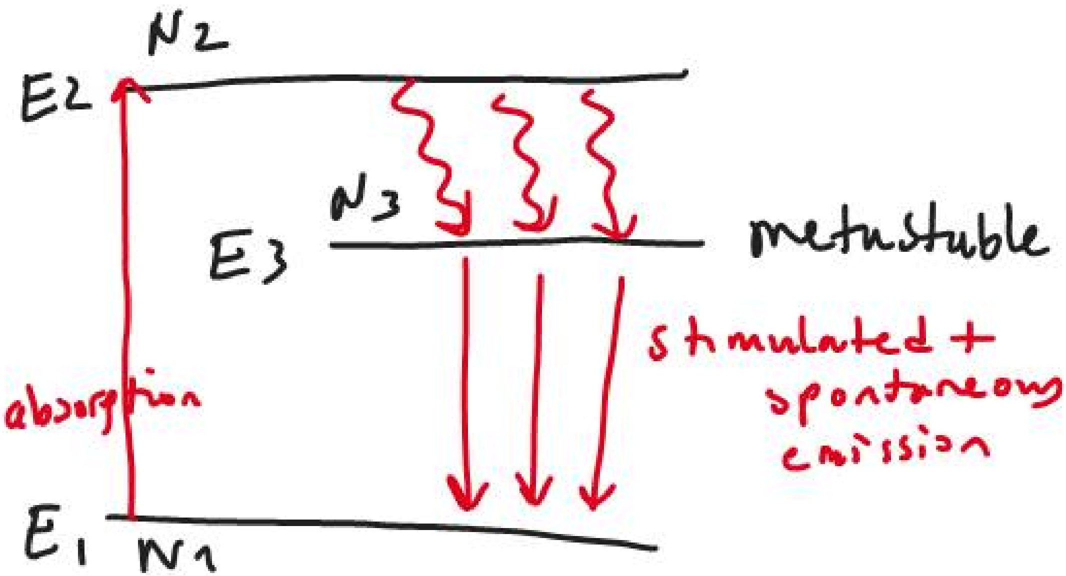
Irregular spacing due to the mode hopping



regular spiking

Nd:YAG

Three-level Lasers



eg: Ruby Laser

RATE EQUATIONS for 3-level systems

$N_1 + N_3 \cong N_t$ (N_2 is small because electrons do not stay at that energy long)

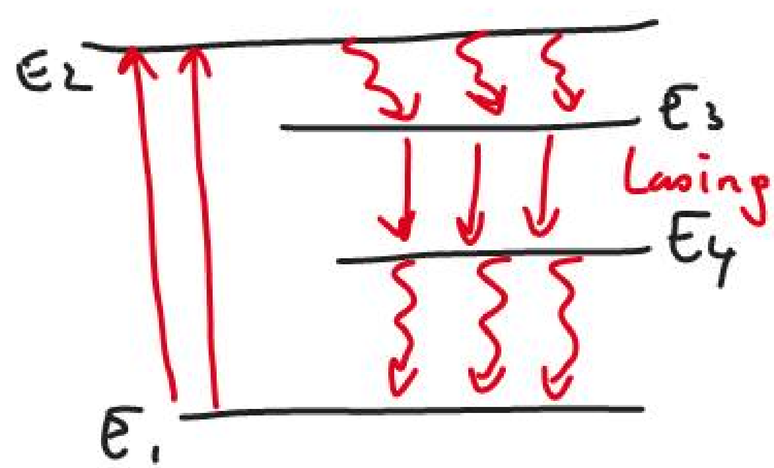
4-level systems:

Nd:YAG

CO_2

Dye lasers

He-Ne laser



$$\frac{dN_3}{dt} = \underbrace{W_p \cdot N_1}_{\text{pump rate (absorption)}} - \underbrace{V \cdot B \cdot p\,(N_3 - N_1)}_{\text{Stimulated emission}} - \underbrace{\frac{N_3}{\tau_{34}}}_{\text{spontaneous emission}}$$

V: Volume of the active region
p: # of photons in the cavity
τ_{34} : lifetime of the carriers in E_3
$B = B_{12} = B_{21}$ Einstein coefficient

$$\frac{dp}{dt} = V B p (N_3 - N_1) - \frac{p}{\tau_c}$$

τ_c : is the cavity lifetime of the photon

$$N_3 - \cancel{N_1} = \Delta N$$
$$N_3 + \cancel{N_1} = Nt$$
$$\overline{N_3 = \frac{Nt + \Delta N}{2}}$$

$$Nt - \Delta N = Nt - (N_3 - N_1)$$
$$= \cancel{N_3} + N_1 - \cancel{N_3} + N_1$$
$$Nt - \Delta N = 2N_1$$
$$N_1 = \frac{Nt - \Delta N}{2}$$

$$\frac{dN_3}{dt} = \frac{1}{2}\frac{d}{dt}(N_t + \Delta N) = \frac{1}{2}\left(\cancel{\frac{dN_t}{dt}} + \frac{d\Delta N}{dt}\right)$$

0

Total states do not change

$$\frac{dN_3}{dt} = \frac{1}{2}\frac{d\Delta N}{dt}$$

$$\frac{1}{2}\frac{d\Delta N}{dt} = W_p N_1 - B \cdot p \cdot V \cdot \Delta N - \frac{N_3}{\tau_{31}}$$

$$\frac{d\Delta N}{dt} = 2 W_p N_1 - 2 B \cdot p \cdot V \cdot \Delta N - \frac{\cancel{2}(N_t + \Delta N)}{\cancel{2}\tau_{31}}$$

$$\frac{d\Delta N}{dt} = \cancel{2}\frac{W_p (N_t - \Delta N)}{\cancel{2}} - 2 B \cdot p \cdot V \cdot \Delta N - \frac{N_t + \Delta N}{\tau_{31}}$$

$$\boxed{\frac{d\Delta N}{dt} = W_p (N_t - \Delta N) - 2 \cdot B \cdot p \cdot V \cdot \Delta N - \frac{N_t + \Delta N}{\tau_{31}}} \quad (1)$$

$$\frac{dp}{dt} = V \cdot B \cdot p \, \Delta N - \frac{p}{\tau_c}$$

$$\frac{dp}{dt} = p \left(V \cdot B \cdot \Delta N - \frac{1}{\tau_c} \right)$$

---

it tells lasing begins when $\frac{dp}{dt} = 0$

$$V \cdot B \cdot \Delta N = \frac{1}{\tau_c}$$

$$\boxed{\Delta N_c = \frac{1}{V \cdot B \cdot \tau_c}}$$

critical value of population inversion ($\Delta N = N_3 - N_1$)

for $\Delta N > \Delta N_c \rightarrow \frac{dp}{dt} > 0$ amplification begins.

A critical pump rate is obtained from (1) by setting $\frac{d \Delta N}{dt} = 0$, $\Delta N = N_c$, and $p = 0$

$$W_p(N_t - \Delta N_c) - 0 - \frac{\Delta N_c + N_t}{\tau_{31}} = 0$$

$$\boxed{W_p = \frac{\Delta N_c + N_t}{\tau_{31}(N_t - \Delta N_c)}}$$

$$\boxed{W_p = \frac{N_{3c}}{N_{1c}} \cdot \frac{1}{\tau_{31}}}$$

$$\boxed{\underbrace{W_p \cdot N_{1c}}_{\text{pump rate}} = \underbrace{\frac{N_{3c}}{\tau_{31}}}_{\text{spontaneous emission rate}}} \Longrightarrow$$

The rate of pumped transitions equals the rate of spontaneous transitions at the critical pump rate for the sustained lasing action.

actually $N_{1c} \approx N_{3c}$

$$\boxed{W_p \approx \frac{1}{\tau_{31}}}$$

A steady-state condition for a constant pump rate that exceeds the critical pump rate will be reached when

$$\frac{d\rho}{dt} = 0 \quad \text{and} \quad \frac{d\Delta N}{dt} = 0$$

This results in

$$\Delta N = \frac{1}{V \cdot B \cdot \tau_c} = \Delta N_c$$

$$p = \frac{V \tau_c}{2} \left[W_p (N_t - \Delta N) \right]$$

is exactly the same as the critical inversion.

Pumping

optical way

Flash lamps are used

Nd: YAG
Nd: Glass
Ruby
⋮

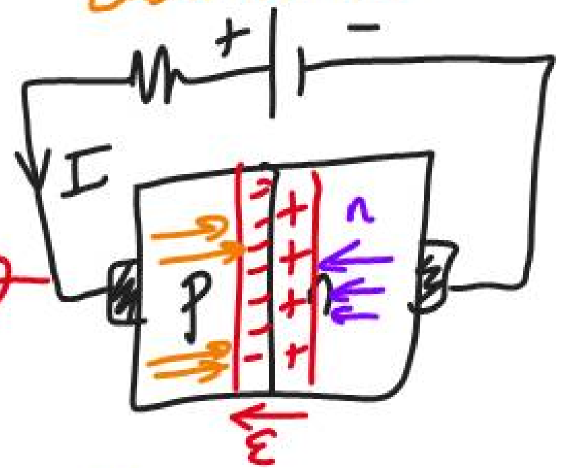
} Solid glass Lasers

+

Dye lasers

electrical way

Laser diode

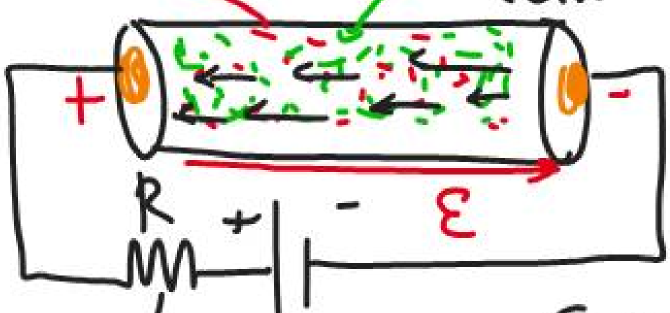


Electrical Discharge

GAS lasers

Lasing gas atoms

Gas atoms/molecules. (discharging)



to control current

High voltage

$$\mathcal{E} = \frac{V}{d}$$

For example: in CO_2 laser
Gas mixture = CO_2, N_2, He
Ionized (discharged) = N_2
Lasing: CO_2

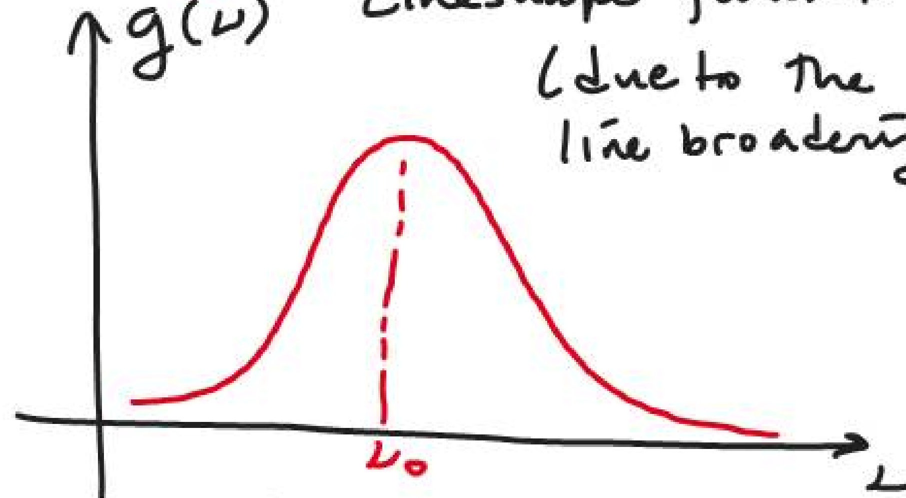
Electrical injection

in semiconductor lasers.

LINE BROADENING

a) Natural Broadening

b) Homogenous & Heterogenous Broadening



Lineshape function (due to the line broadening)

$$\int_0^\infty g(\nu)\,d\nu = 1$$

Normalized

$$I \cdot g(\nu) = I(\nu)$$

NATURAL BROADENING

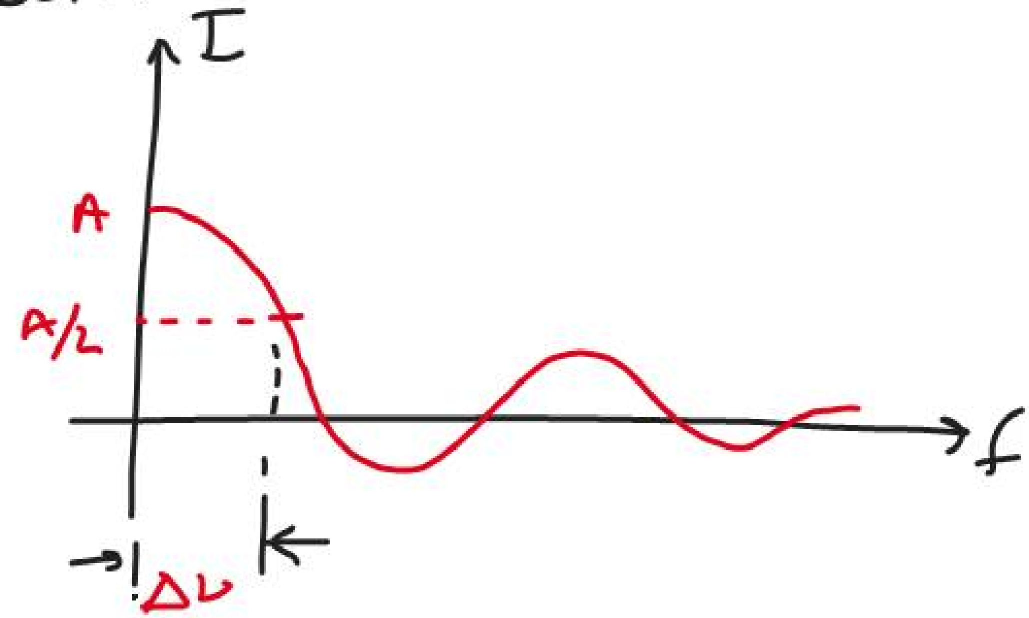
When an atom or molecule is excited to a higher state it does not emit photon instantly and return to the lower energy state.

NATURAL BROADENING

finite time for the atom/molecule to return to the lower energy state

τ : lifetime $\approx 10^{-8}$ sec.

$$\boxed{\Delta\nu \cdot \tau = 1}$$



Full width at half maximum

$\Delta\nu = FWHM$

Broadening in frequency

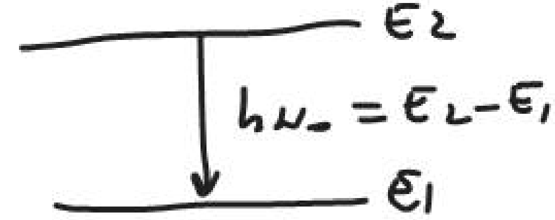
Homogenous Broadening

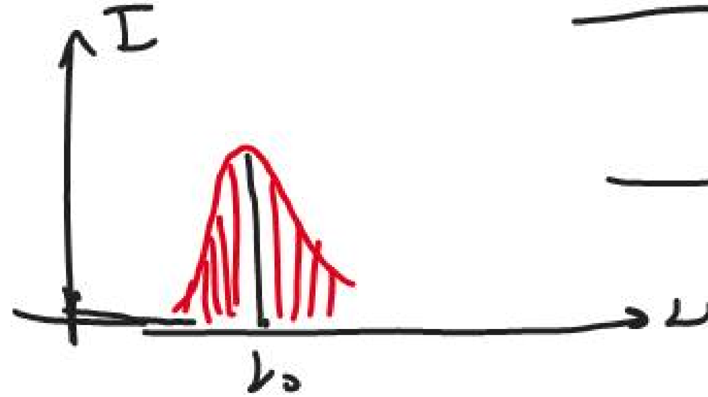
due to the collisions between lasing atoms/molecules

Lineshape function is Lorentzian

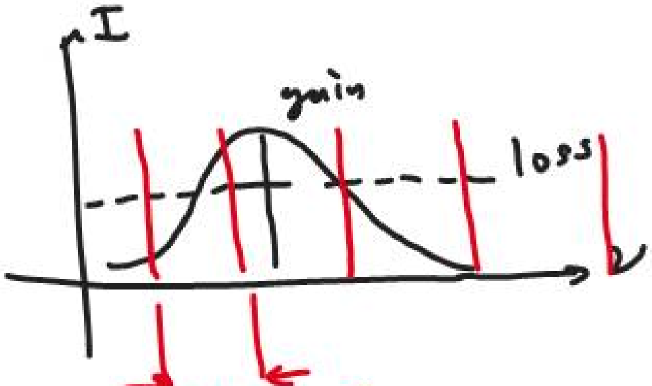
$$\Delta\nu \cdot t_c = 1$$

t_c : meantime between collisions.





$E = h\nu_0$



$$\nu_F = \frac{c}{2d}$$

$10^{14} - 9\times10^{9}$ $\quad 10^{14}$

$10^{14} + 9\times10^{9} \approx 10^{14}$

Inhomogenous Broadening

Lineshape function is Gaussian

- Doppler effect in gas lasers

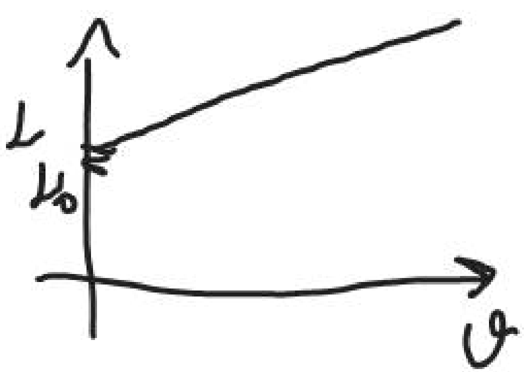
$$\nu = \frac{\left(1+\frac{u}{c}\right)}{\sqrt{1-\frac{u^2}{c^2}}} \cdot \nu_0$$

ν_0 = Frequency measured by an observer at rest relative to emitter

u : velocity of the gas atom/molecule

$\frac{u}{c} << 1$

$$\nu \cong \left(1+\frac{u}{c}\right)\nu_0$$



A lineshape function represents these line broadening effect.

$$\beta(\nu) = \frac{(N_2 - N_1)\cdot c^2}{8\pi\nu^2}\cdot g(\nu) \qquad \int_0^\infty g(\nu)\,d\nu = 1$$

β takes the shape of $g(\nu)$.