06.11.2012

Celal Zaim Çil ©

The Photon

A photon carries
- EM energy
- momentum (linear)
- intrinsic angular momentum (spin associated with the polarization process)
- orbital angular momentum (associated with the wave's spatial distribution)

has **ZERO REST MASS**

and travels with the speed of light (c) in vacuum.

Photon Energy

$$E = h \cdot \nu \qquad 6.63 \times 10^{-34}\ J \cdot s$$

ν: frequency h: the Planck's constant

$\omega = 2\pi\nu \longrightarrow \nu = \dfrac{\omega}{2\pi}$

$E = h \cdot \dfrac{\omega}{2\pi} = \dfrac{h}{2\pi} \cdot \omega$

$\boxed{E = \hbar\omega}$ $\qquad \hbar = \dfrac{h}{2\pi}$

$\boxed{E = \hbar\omega}$

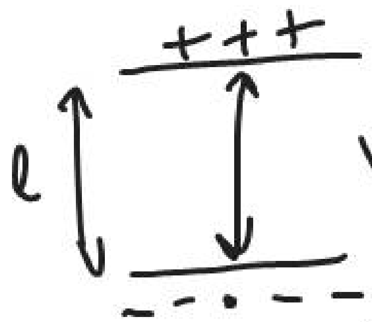
<u>ex</u>: IR wavelength $\lambda_0 = 1\,\mu m$ in free space

$$\nu_0 = \frac{c_0}{\lambda_0} = \frac{3\times10^{8}\,m/s}{1\times10^{-6}\,m} = 3\times10^{14}\,Hz$$

$$E = h\cdot\nu = 6.63\times10^{-34}\,J\cdot\cancel{s} \times 3\times10^{14}/\cancel{s} = 1.99\times10^{-19}\,J$$

$1\,eV = 1.66\times10^{-19}\,J$

$$E = \frac{1.99\times10^{-19}\,\cancel{J}}{1.66\times10^{-19}\,\cancel{J}/eV} = \underline{1.24\,eV}$$



$V = 1.24\ V$ The energy given to an electron by accelerating it with $1.24V \Rightarrow 1.24\,eV$ in vacuum.

$\mathcal{E} = \dfrac{V}{l}$

$$\boxed{E = q.V}$$

$q = 1.6\times10^{-19}\ C$ (charge of an electron)

$$\boxed{E\,(eV) = \frac{1.24}{\lambda_0\,(\mu m)}}$$

$\lambda_0 = 1\mu m \rightarrow E = 1.24\,eV$

$\lambda_0' = 0.1\mu m \rightarrow E' = 12.4\,eV$

$\lambda_0'' = 1cm \rightarrow E'' = \dfrac{1.24}{10^4\,\mu m} = 1.24\times10^{-4}\,eV$

Photon Position

We have a wave associated with a photon described by a complex wavefunction:

$$U(\vec{r})\exp(j2\pi\nu t) \quad \text{of the mode.}$$

The probability of observing a photon at a point $\vec{r}$ within an incremental area $d\bar{A}$ at any time is proportional to the local optical intensity

$$I(\vec{r}) \propto |U(\vec{r})|^2$$

$$P(\vec{r})\,d\vec{A} \propto I(\vec{r})\,d\bar{A}$$

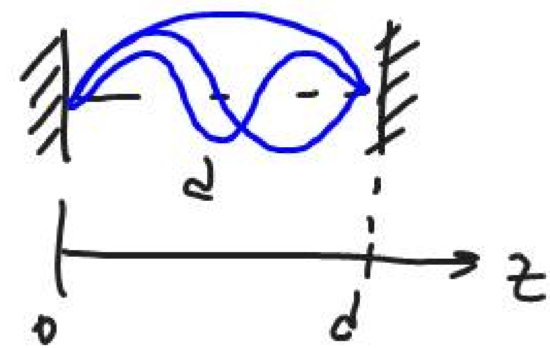
Photon is most probably found where the intensity is high

A photon in a mode (for instance in a mode in a resonator)

is described by a standing wave
with the intensity distribution

$\underline{I}(x, y, z) \propto \sin(\pi z/d)$

$\left(\nu_F = \frac{c}{2d} \right.$
Freq. separation
between the modes)

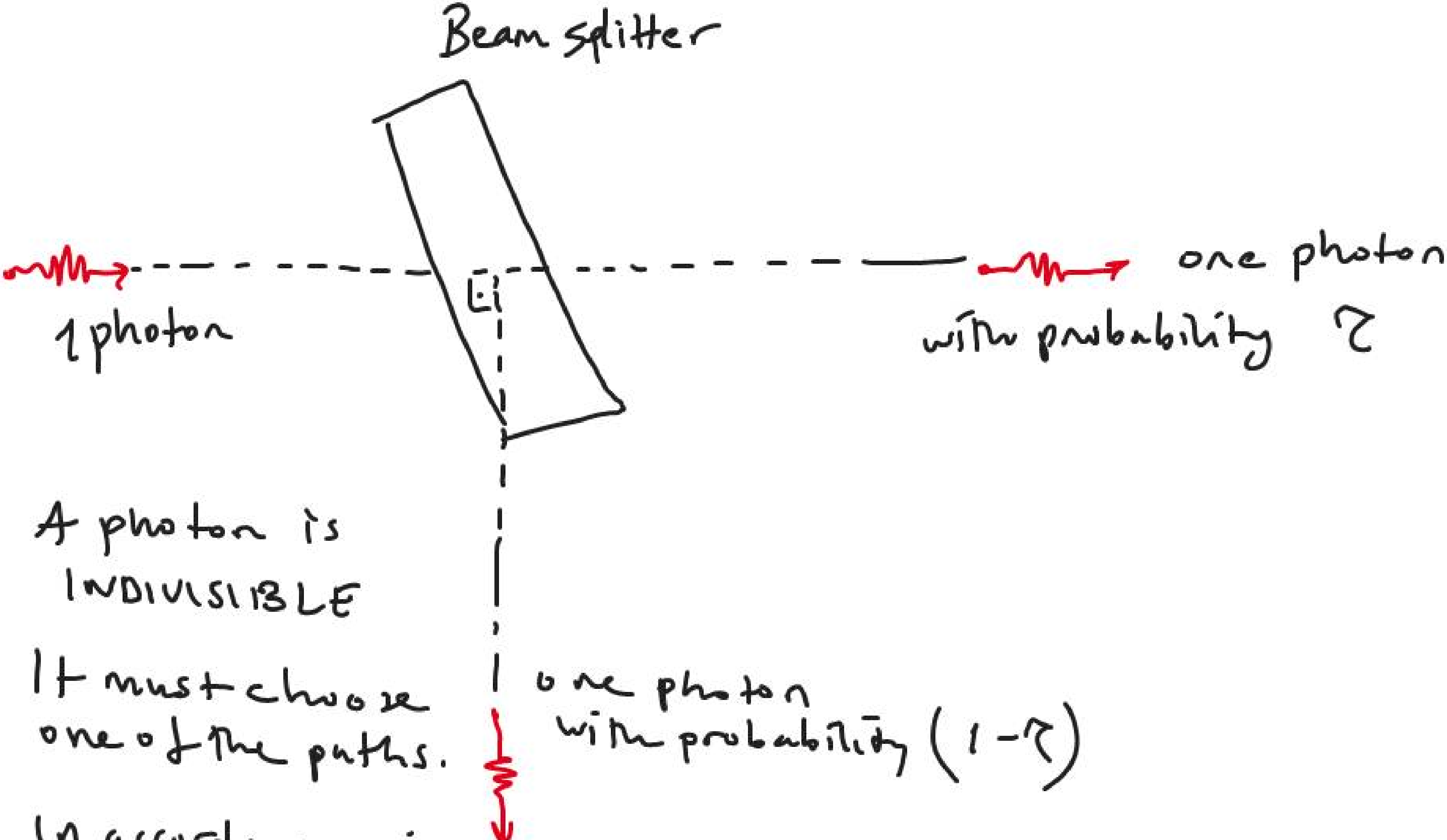


it's most likely to be detected

at $z = \frac{d}{2}$, $\sin(\pi/2) = 1$

In contrast to the waves, which are extended in space
and the particles, which are localised

optical photons behave as extended and localised entities.
This behavior is called WAVE-PARTICLE DUALITY.



A photon is INDIVISIBLE

It must choose one of the paths.

In accordance with the photon position rule $p(\vec{r})\cdot d\vec{A} \propto I(\vec{r})\, d\vec{A}$

Photon Momentum

In Classical electromagnetic optics, an electromagnetic plane wave carries a linear momentum density (per unit Volume) $\Rightarrow \left(\frac{W}{c} \cdot \vec{k}\right)$ W: energy density (J/m³)

k : wave vector $= \frac{2\pi}{\lambda}$

In Photon optics the linear momentum of a photon

$$\bar{p} = \left(\frac{E}{c}\right) \cdot \vec{k} \qquad E = h \cdot \nu \quad \text{photon energy}$$

The linear momentum associated with the plane wave mode of wave vector $\vec{k}$

$$\boxed{\vec{p} = \hbar \cdot \vec{k}}$$

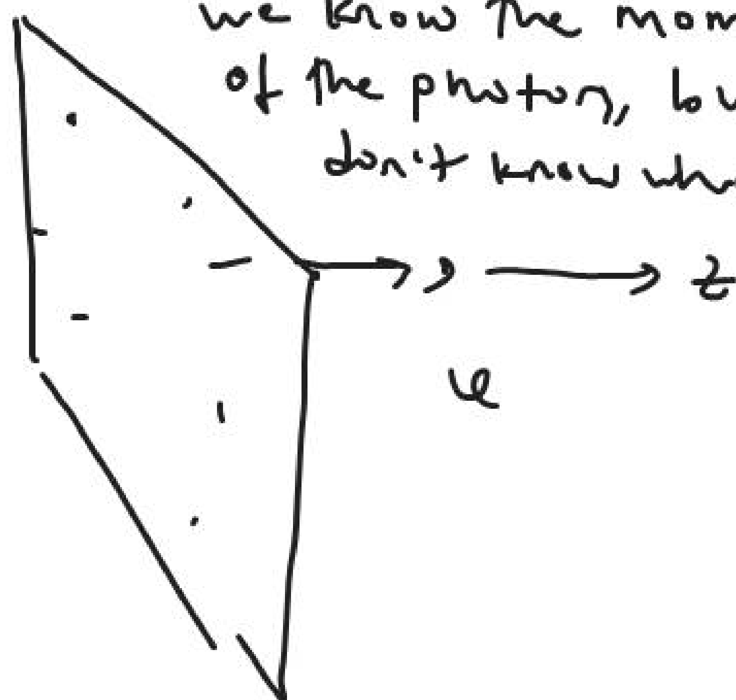
$$\frac{E}{c} = \frac{h \cdot \nu}{c} = \frac{h \cdot \frac{\cancel{c}}{\lambda}}{\cancel{c}} = \frac{h}{\lambda}$$

$$\frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda} = \underbrace{\left(\frac{2\pi}{\lambda}\right)}_{k}\hbar$$

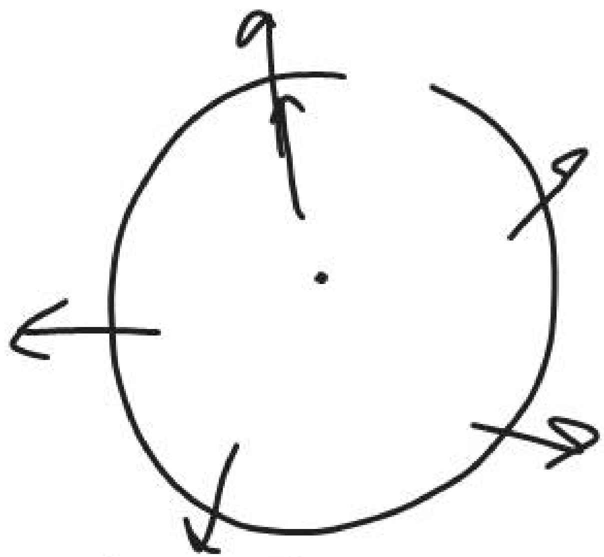
de Broglie

$$\boxed{p = \frac{h}{\lambda} = \hbar k}$$

In a plane wave



we know the momentum of the photon, but we don't know where the photon is



In a spherical wave we know where the photon is (center) but we have no idea about its momentum.

$$\left.\begin{aligned} \Delta E \cdot \Delta t &\geq \frac{\hbar}{2} \\ \Delta p \cdot \Delta x &\geq \frac{\hbar}{2} \end{aligned}\right\} \text{Heisenberg's uncertainty principle}$$

Photon Spin Angular Momentum

associated with circularly polarised states.

The magnitude of photon spin is quantized as

$$S = \pm \hbar$$

$+$ right-handed ($\circlearrowright$)

$-$ left-handed ($\circlearrowleft$)

In the same way photons transfer linear momentum to an object, circularly polarised photons can exert a TORQUE on the object.

Photon orbital Angular Momentum

associated with its spatial distribution

* A Laguerre Gaussian beam has an azimuthal phase dependence $\propto \exp(j l \varphi)$, and therefore a helical wavefront
has an angular momentum for $l \neq 0$.
A photon in such a spatial mode possesses an angular momentum

$$\boxed{L = l \cdot \hbar}$$