PLANAR MIRROR RESONATORS

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09.10.2012
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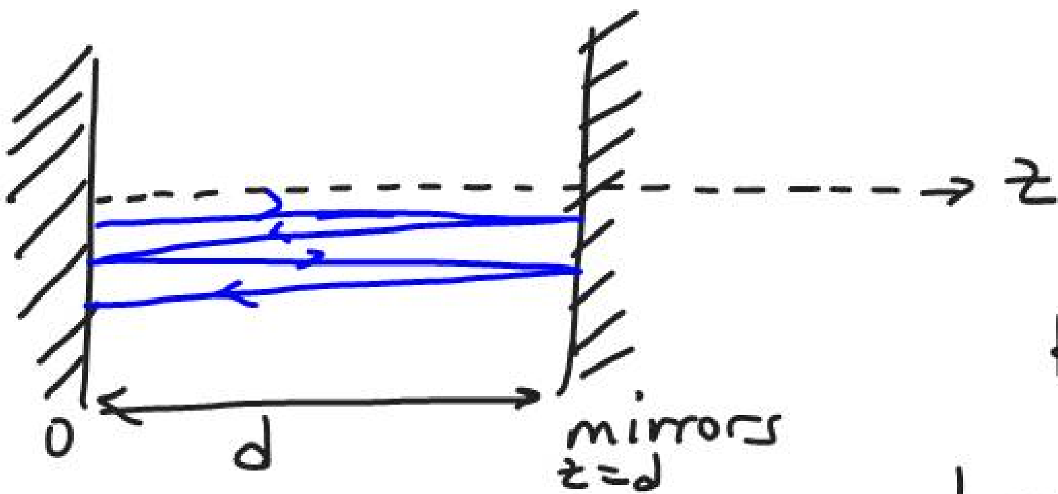


a Fabry-Perot Resonator

$u(\vec{r}) = A_r \sin kz$

$u(\vec{r}) = 0$ at $z=0$ and d

$\sin kz = 0$ at $z=d$

$kd = q\pi$

$q = 1, 2, 3, \ldots$

(1/m)

$k = \frac{\pi}{d} q \qquad q = 1, 2, 3, \ldots$

k: wave number is quantized.

$k = \frac{2\pi}{\lambda} \qquad \lambda$: wavelength

$\frac{2\pi}{\lambda} = \frac{\pi}{d} q$

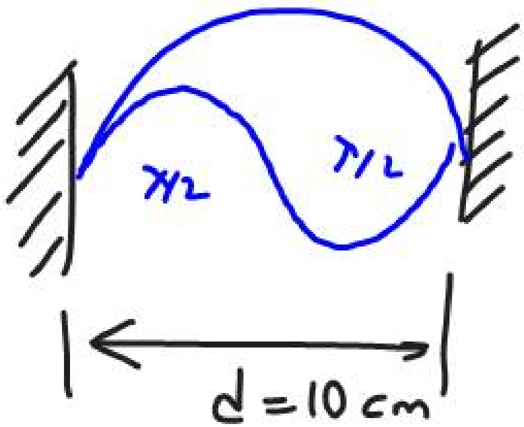
$$\boxed{\lambda = \frac{2d}{q}}$$

$$\frac{\lambda}{2} = \frac{d}{q}$$

$$c = \lambda \cdot \nu$$

ν: frequency (Hz)

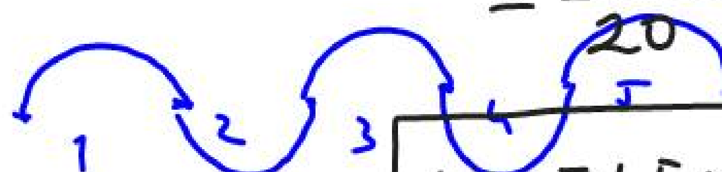
$$\nu = \frac{c}{\lambda} = \frac{c}{2d} \cdot q$$



$$\boxed{\nu_F = \frac{c}{2d} \cdot q}$$

$$\frac{3\times10^8 \text{ m/s}}{2\times10\times10^{-2}} \times 1$$

$$= \frac{3\times10^{10}}{20} = \frac{3\times10^9}{2}$$



$$\boxed{\nu_F = 1.5\times10^9 \text{ Hz}} \quad q = 1$$

Frequency separation in the Resonator

Modes

$\frac{\lambda}{2} = 10\text{cm}$ for $q = 1$

$\frac{\lambda}{2} = 5\text{cm}$ for $q = 2$

$\frac{\lambda}{2} = \frac{10}{3}$ no $q = 3$ possible

$\frac{\lambda}{2} = \frac{10}{4}$ no $q = 4$ possible

$\frac{\lambda}{2} = \frac{10}{5} = 2\text{cm}$ for $q = 5$

2mm

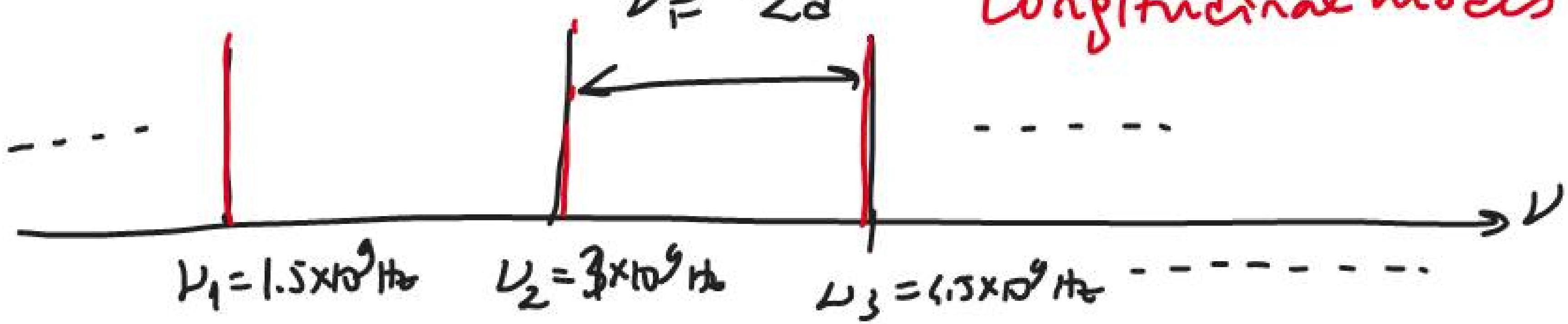
$\frac{\lambda}{2} = \frac{100\text{ mm}}{50}$ $\quad q = 50$

$\frac{\lambda}{2} = \frac{100\,000\,\mu\text{m}}{100}$ $\quad q = 100$

$\frac{\lambda}{2} = 1000\,\mu\text{m} = 1\text{ mm}$

⋮

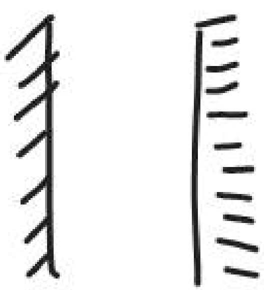
Resonator modes:



Longitudinal modes

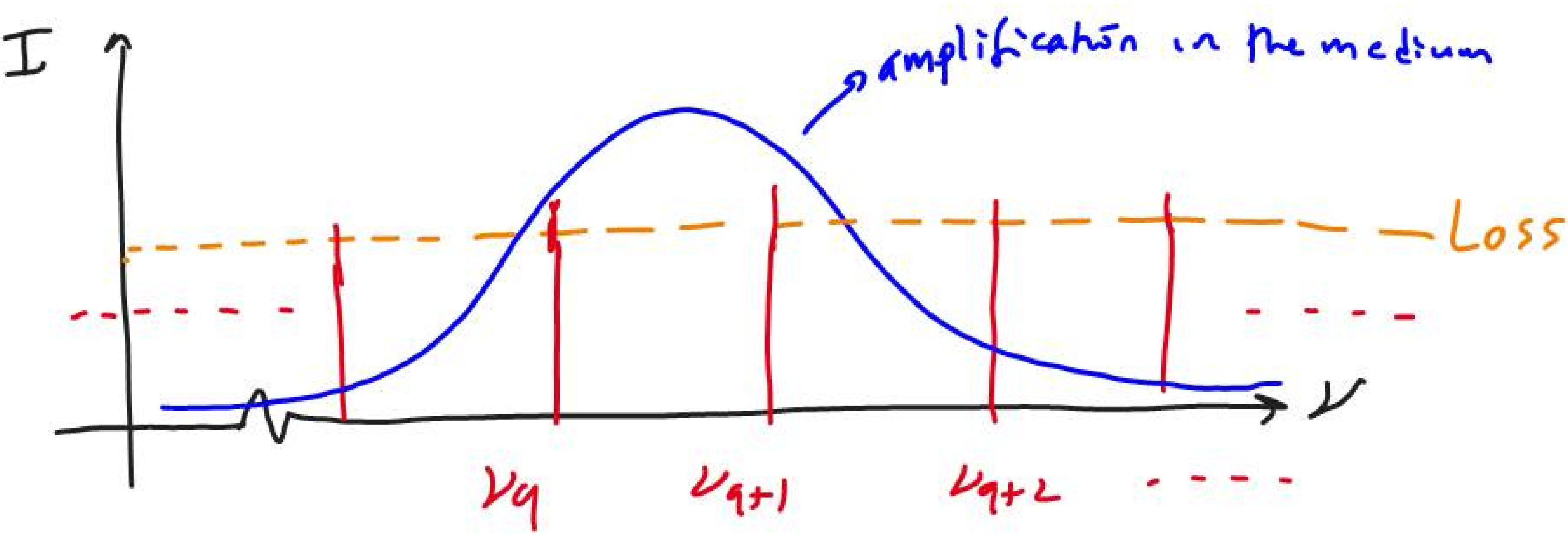
Longitudinal modes

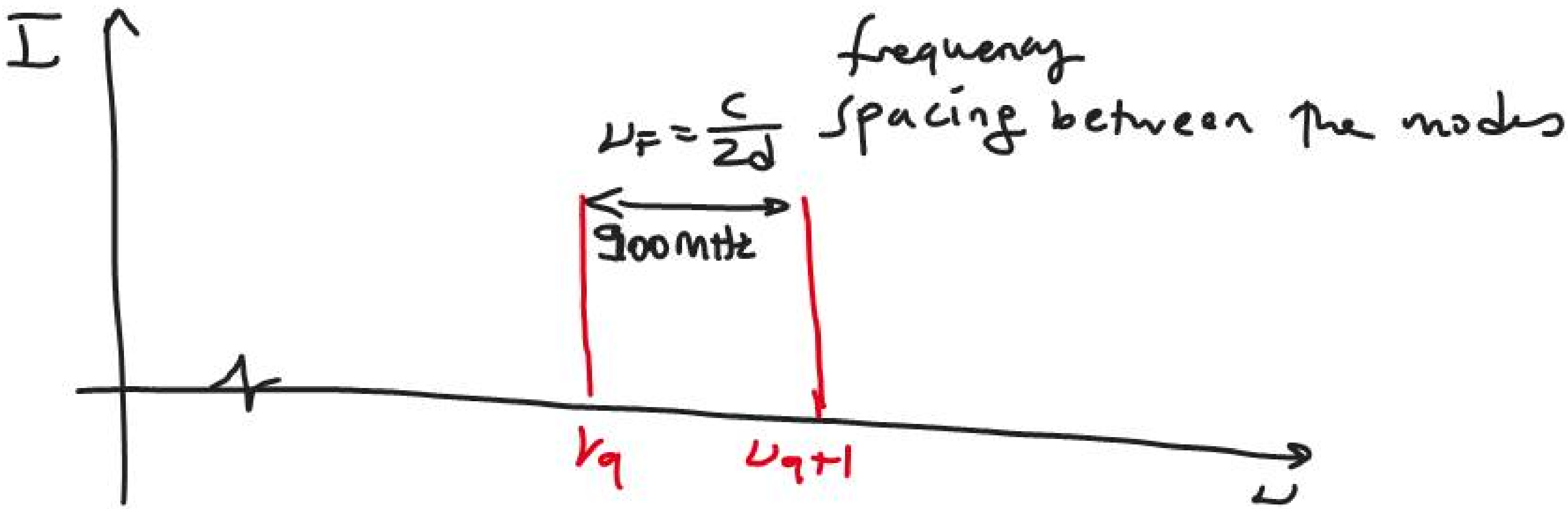
Infinite number of modes available



d = 1.5 cm

$$\nu_F = \frac{3\times10^8 \text{ m/s}}{2\times1.5\times10^{-2}\text{ m}} = \frac{3}{3}\times10^{10} = 10^{10} \text{ Hz}$$

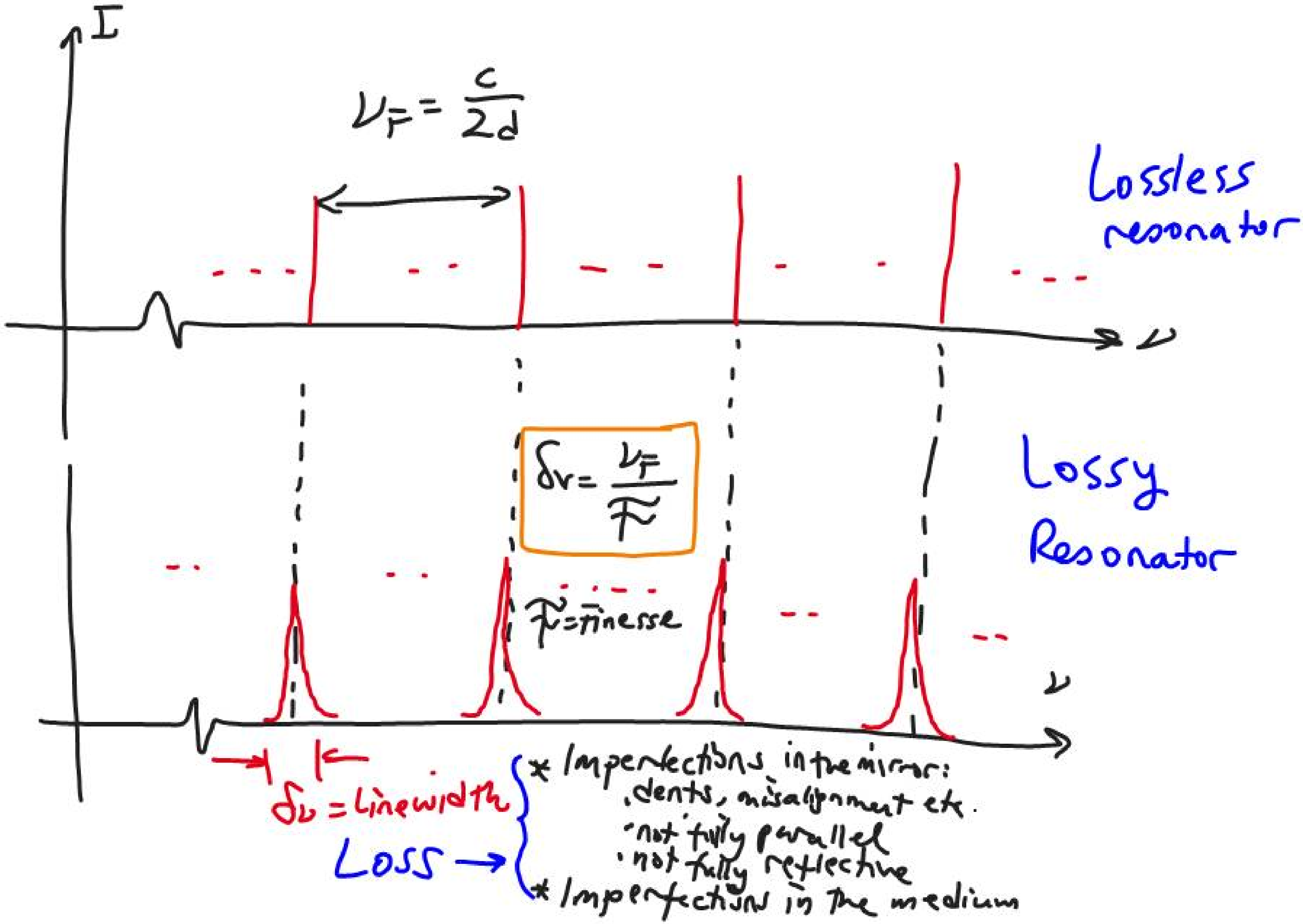




10 THz

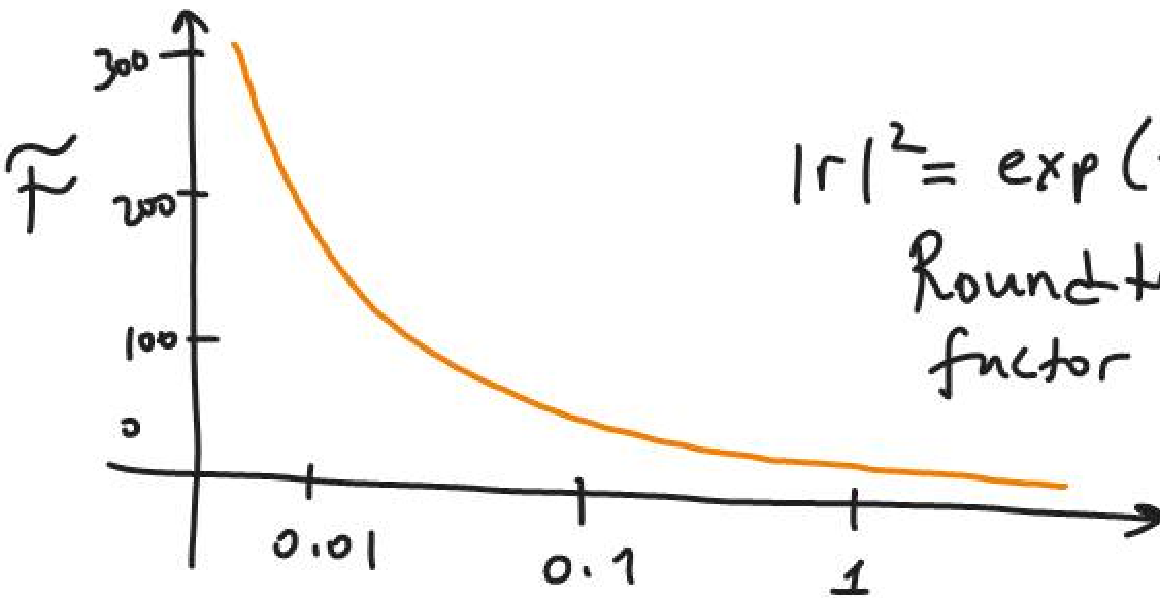
10×10^{12} $\quad$ $10 \times 10^{12} + 9 \times 10^{8}$

Lossless and Lossy Resonator



$\tilde{\mathcal{F}}$: Finesse $= \dfrac{\pi}{\alpha_r \cdot d}$

α_r: loss per unit length (1/m)



$|r|^2 = \exp(-2\alpha_r \cdot d)$

Round trip intensity attenuation factor (2d : round trip)

The higher $\tilde{\mathcal{F}}$ the better $\delta\nu$

$c \cdot \alpha_r$ = loss per unit time

$$\delta\nu = \frac{\nu_F}{\tilde{\mathcal{F}}} = \frac{\frac{c}{2d}}{\frac{\pi}{\alpha_r \cdot d}} = \frac{c \cdot \alpha_r}{2\pi} = \frac{1}{2\pi \tau_p}$$

Characteristic decay time $\boxed{\tau_p = \dfrac{1}{c \cdot \alpha_r}}$

A resonator is generally defined by 2 parameters:

* $\nu_F = \frac{c}{2d}$ Frequency spacing

* $\delta\nu = \frac{\nu_F}{\mathcal{F}} = \frac{c \cdot \alpha_r}{2\pi} = \frac{1}{2\pi \tau_p}$ linewidth

$\tau_p = \frac{1}{c \cdot \alpha_r}$ Decay time, or photon lifetime, or resonator lifetime.

$$\delta\nu \cdot \tau_p = \frac{1}{2\pi \tau_p} \cdot \tau_p = \frac{1}{2\pi}$$

$$\boxed{\delta\nu \cdot \tau_p = \frac{1}{2\pi} \text{ (constant)}}$$

Another expression of the Heisenberg's uncertainty principle.

Quality Factor (Q)

$$Q = 2\pi \frac{\text{stored energy}}{\text{energy loss per cycle}}$$

In a resonator

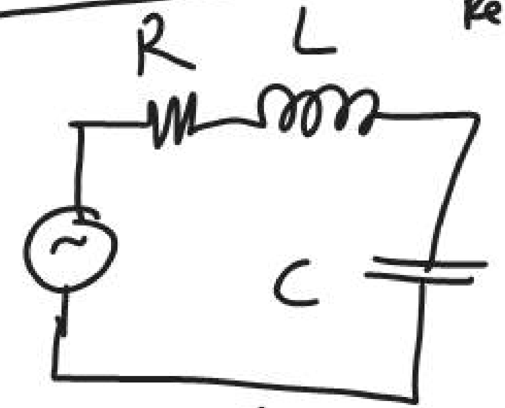
stored energy = E

energy lost at a rate $= \frac{E}{\tau_p} = E \cdot \alpha_r \cdot c$

$\frac{E \, \alpha_r \cdot c}{\omega_0}$: energy lost per cycle

$$Q = 2\pi \frac{E}{\frac{E \, \alpha_r \cdot c}{\omega_0}} = \frac{2\pi\omega_0}{\alpha_r \cdot c}$$

Reminder



$\omega_0 = \frac{1}{2\pi\sqrt{LC}}$

Resonance frequency of the RLC circuit

$Q = \frac{2\pi\omega_0 L}{R}$ above

$$Q = \frac{2\pi \nu_0}{\alpha_r \cdot c}$$

ν_0: resonance frequency of the resonator

$$\delta\nu = \frac{\alpha_r \cdot c}{2\pi}$$

$$Q = \frac{\nu_0}{\delta\nu}$$

Energy stored in the resonator increases as loss decreases

loss $\downarrow \Rightarrow \delta_\nu \downarrow \Rightarrow Q \uparrow$

$$Q = 2\pi \nu_0 \cdot \tau_p$$

SUMMARY:

2 parameters characterize the losses in an optical resonator:

1) The loss coefficient, α_r
2) The photon lifetime, $\tau_p = \frac{1}{\alpha_r \cdot c}$

2 parameters characterise the quality of the optical resonator:

1) Finesse $\mathcal{F} \simeq \frac{\pi}{\alpha_r \cdot d}$
2) The quality factor $Q = 2\pi \nu \tau_p = \frac{\nu}{\delta\nu}$

2 frequencies determine the optical characteristics of an optical resonator:

1) The frequency spacing between the modes $\nu_F = \frac{c}{2d}$
2) The spectral width (linewidth) $\delta\nu = \frac{\nu_F}{\mathcal{F}}$