(from: An introduction to optoelectronics
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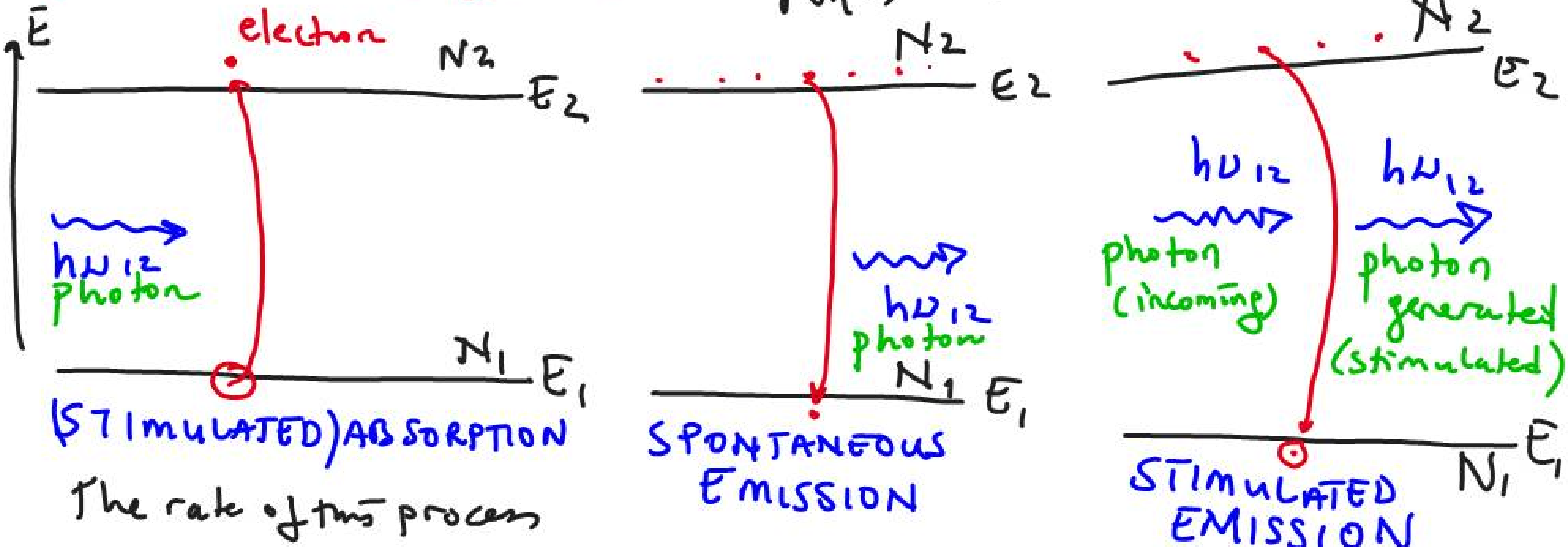
QUANTITAVIE EVALUATION

1. 2-level Systems

$N_2 \Rightarrow$ # of electrons at E_2
$N_1 \Rightarrow$ # of " at E_1



The rate of this process

$$R_{12} = B_{12} \cdot \rho_\nu \cdot N_1 \qquad S_{21} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot \rho_\nu \cdot N_2$$

ρ_ν : density of photons (per volume, per frequency) at energy $h\nu_{12}$

B_{12}, A_{21}, B_{21} : Einstein's coefficients

In equilibrium upward transitions = downward transitions

$$R_{12} = S_{21} + R_{21}$$

$$B_{12} \cdot \rho_\nu \cdot N_1 = A_{21} \cdot N_2 + B_{21} \cdot \rho_\nu \cdot N_2$$

$$B_{12} \rho_\nu = A_{21} \frac{N_2}{N_1} + B_{21} \rho_\nu \frac{N_2}{N_1}$$

$$\rho_\nu \left(B_{12} - B_{21} \cdot \frac{N_2}{N_1} \right) = A_2 \frac{N_2}{N_1}$$

$$\rho_\nu = \frac{A_{21} N_2/N_1}{B_{12} - B_{21} \frac{N_2}{N_1}} = \frac{A_{21} \frac{N_2}{\cancel{N_1}}}{\frac{B_{12} N_1 - B_{21} N_2}{\cancel{N_1}}} = \frac{A_{21}}{B_{12} \frac{N_1}{N_2} - B_{21}} = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1}$$

$$\rho_\nu = \frac{A_{21}/B_{21}}{(B_{12} N_1 / B_{21} N_2) - 1}$$

$$\frac{N_1}{N_2} = \exp\left(-\frac{E_1 - E_2}{kT}\right)$$

$$E_2 - E_1 = h\nu_{12}$$

$$\rho_\nu = \frac{A_{21}/B_{21}}{\left(B_{12}/B_{21}\right)\left(\exp\frac{h\nu_{12}}{kT} - 1\right)}$$

For Blackbody radiation at equilibrium

$$\rho_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\left(\exp\frac{h\nu}{kT} - 1\right)}$$

$$B_{12} = B_{21}$$

$$A_{21} = B_{21}\cdot\frac{8\pi h\nu^3}{c^3}$$

EINSTEIN'S RELATIONS

Results:

1) The ratio of stimulated to spontaneous emission

$$S = \frac{R_{21}}{S_{21}} = \frac{\rho_\nu \cdot B_{21} \cdot \cancel{N_2}}{A_{21} \cdot \cancel{N_2}}$$

$$S = \rho_\nu \cdot \frac{B_{21}}{A_{21}}$$

$$S = \frac{1}{\left(\exp\frac{h\nu}{kT} - 1\right)}$$

$$S = \rho_\nu \cdot \frac{c^3}{8\pi h \nu^3}$$

$$S = \frac{\cancel{8\pi h \nu^3}}{\cancel{c^3}} \cdot \frac{\cancel{c^3}}{\cancel{8\pi h \nu^3}} \cdot \frac{1}{\left(\exp\frac{h\nu}{kT} - 1\right)}$$

$$S = \frac{1}{\exp\frac{h\nu}{kT} - 1}$$

for He-Ne discharge at $T = 370\,K$

$\lambda = 632.8\,nm$

$\nu = 4.74 \times 10^{14}\,Hz$

$$S = 2 \times 10^{-27}$$

Stimulated Emission is very unlikely to occur in a 2-level system in equilibrium.

Mostly spontaneous emission occurs.

2) $$B_{21} = A_{21} \cdot \frac{c^3}{8\pi h \nu^3}$$

This means it is harder to obtain stimulated emission at higher frequencies.

stimulated ← $$\frac{B_{21}}{A_{21}} = \frac{c^3}{8\pi h \nu^3}$$ ← spontaneous

$$\frac{B_{21}}{A_{21}} \propto \nu^{-3}$$

IR Laser → Easier to make

Blue Laser → Difficult to make

Light Amplification by Stimulated Emission of Radiation

LASER

Microwave Amplification " " "

MASER

First MASERS were invented. Then LASERS " "

Since it is easier to make MASERS than LASERS ($\nu_{maser} < \nu_{laser}$)

ν_{maser} → Radio frequency $\sim 10^9$ Hz

ν_{laser} → Light frequencies $\sim 10^{14}$ Hz

In order to have lasing action we have to disturb the equilibrium and make $\frac{N_2}{N_1} > 1$, and to increase ρ_ω as well:

$$N_2 \rho_\nu B_{21} \gg N_2 A_1$$

$$\rho_\nu \gg \frac{A_{21}}{B_{21}}$$

$$\rho_\nu \gg \frac{8\pi h \omega^3}{c^3}$$

However, increasing ρ_ν also increases the absorption when ρ_ω gets large

$$N_1 \cdot \rho_\nu B_{12} \simeq N_2 \rho_\nu B_{21}$$

ABSORPTION STIMULATED EMISSION

In 2-Level system:

for $B_{12} \approx B_{21}$

$$N_1 = N_2$$

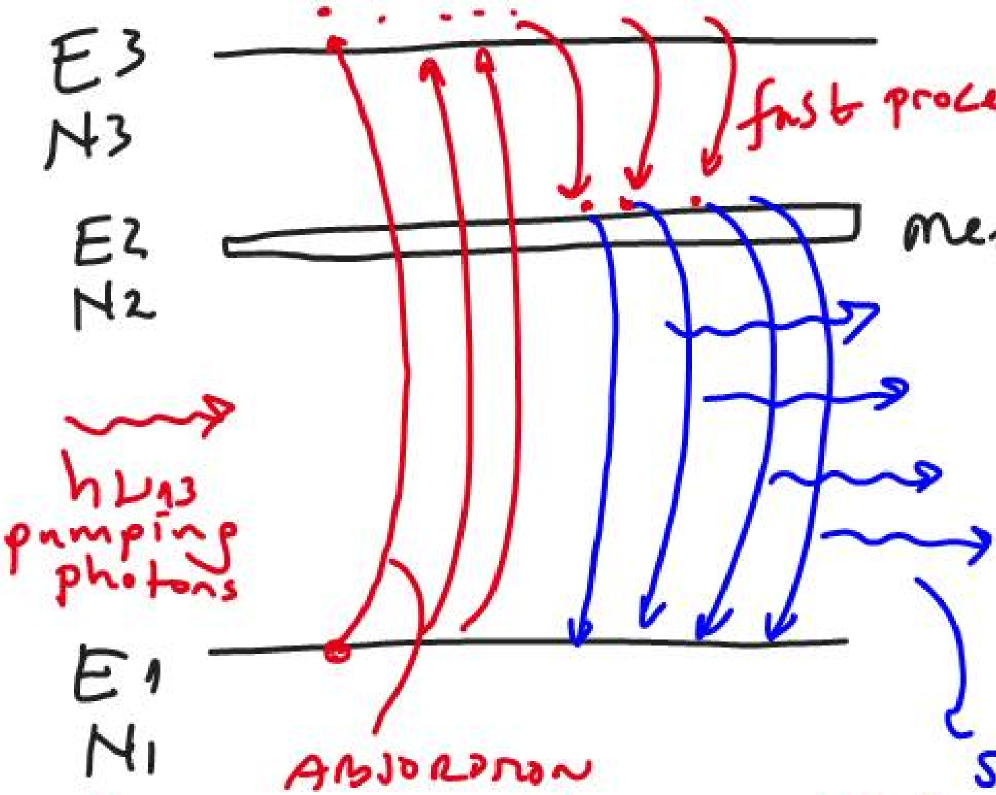
At most N_2 can be equal to N_1, but cannot be bigger than N_1.

Hence, we can not increase N_2 higher than N_1 by increasing the value of the photon field ($\rho\nu$) by pumping the light (photons). Because, these photons are used for absorption as well

POPULATION INVERSION can not be obtained.

Therefore it cannot LASE.

2) A 3-LEVEL SYSTEM



The system must be COOLED.

If we increase pumping to a level then we can have a situation such that

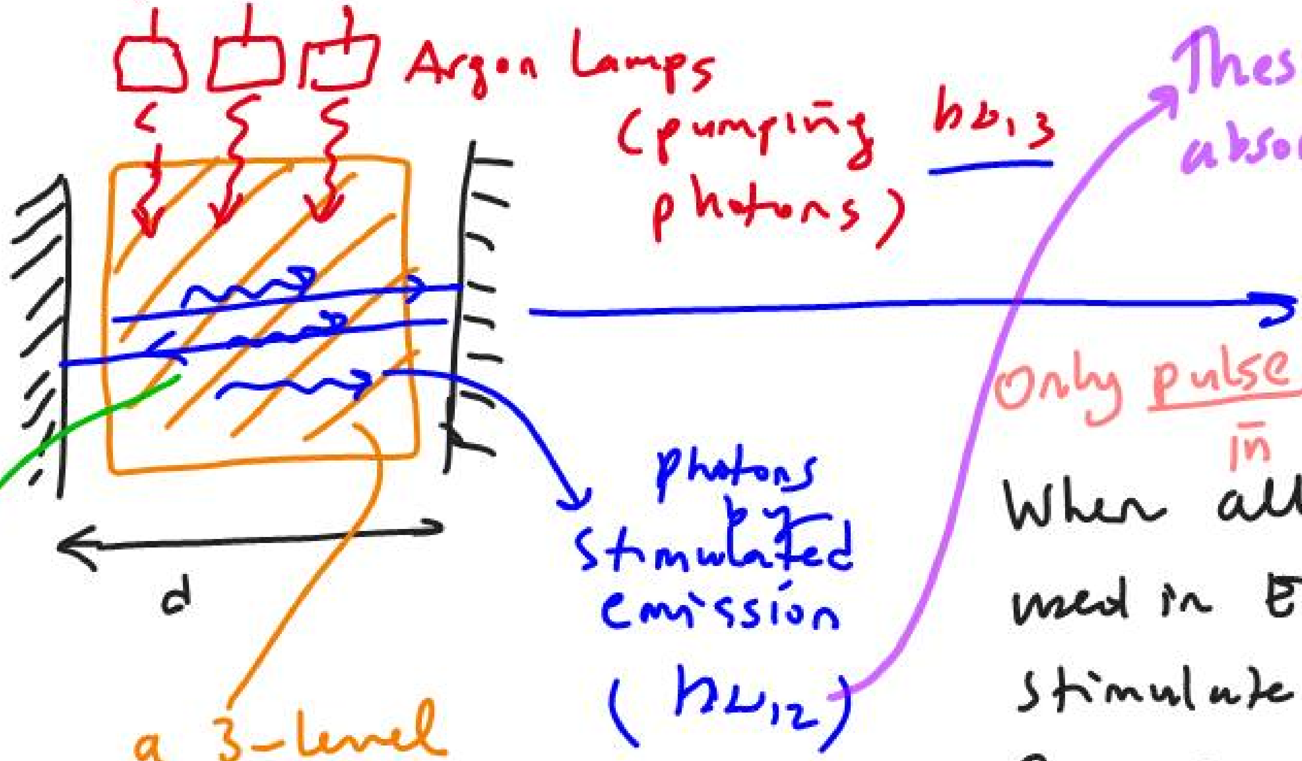
$$N_2 > N_1$$

population inversion to LASE is possible.

$h\nu_{13}$ is absorbed by the material and the electron is raised to E_3.

Stimulated emission $h\nu_{12}$ } These photons are not absorbed by the material and can be amplified by a Resonator.

Photons used for ABSORPTION are not these PHOTONS ($h\nu_{12}$)
($h\nu_{13}$)



These photons are not absorbed again ($h\nu_{12} < h\nu_{13}$)

stimulated photons are amplified by the resonator

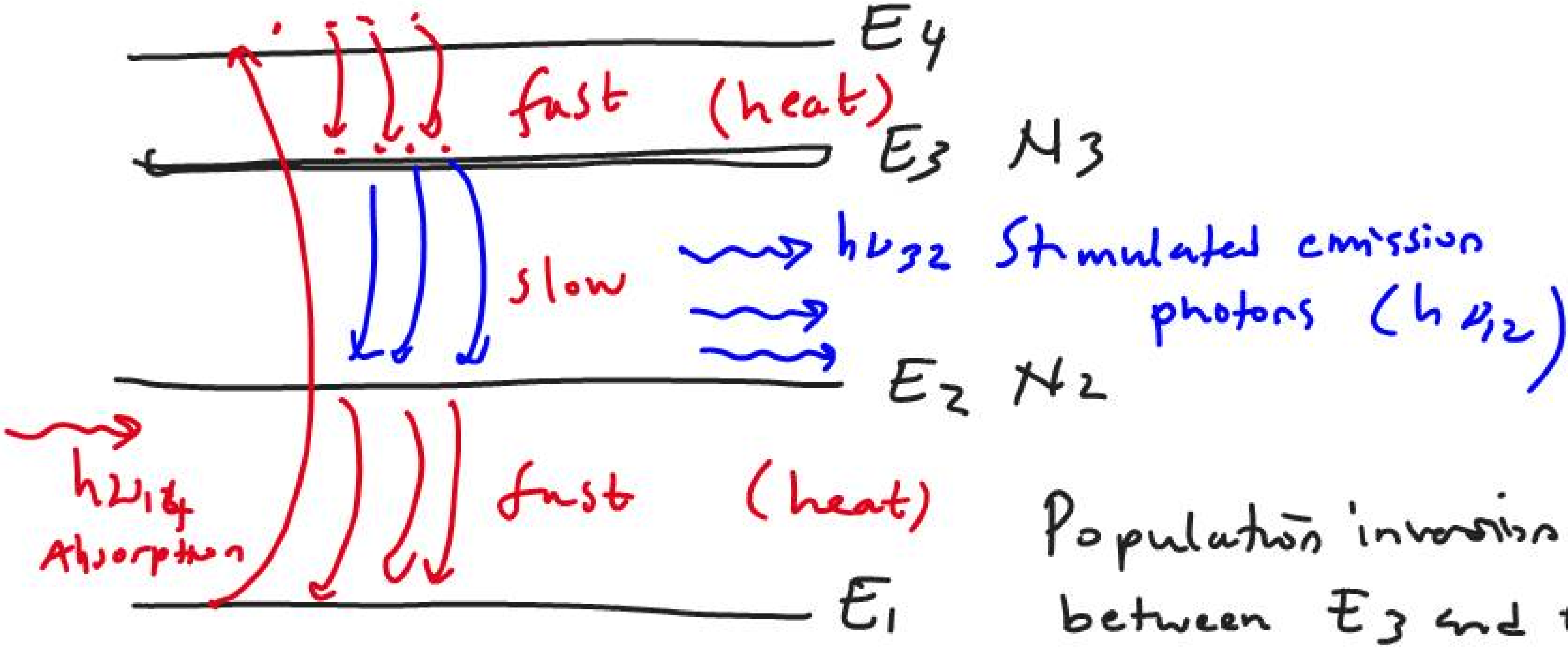
Only pulse operation is possible in 3-level LASER:

When all electrons are used in E_2 to emit stimulated photons Then population inversion is no longer sustained.

Therefore a pulse of Laser is obtained by the stimulated emission photons. The other pulse can only be generated when pumping and absorption populates E_2 level of energy with enough electrons such that N_2 becomes greater than N_1 ($N_2 > N_1$)

i.e., population inversion is obtained again.

3- A 4-level Laser



Nd: Glass Laser

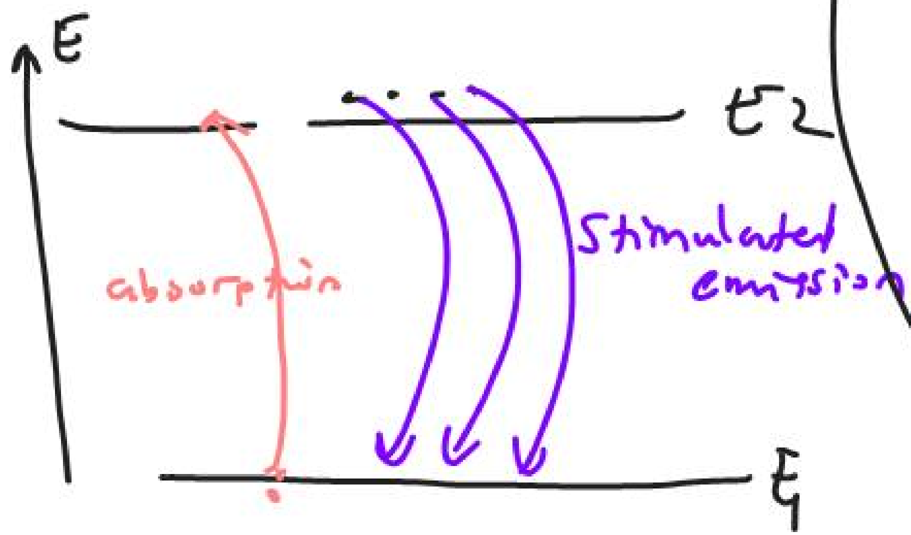
He-Ne Laser

Population inversion takes place between E_3 and E_2, ($N_3 > N_2$) and can be continuously and easily sustained.

Therefore, a continuous wave (CW) operation is possible in these lasers.

If we have a <u>Lasing system</u> between energy levels of E2 and E1:

The # of photons per unit volume (stimulated emission and absorption) that are entering the system is p_{12}.



$$\frac{dp_{12}}{dt} = \underbrace{N_2 \rho_{\nu_{12}} B_{21}}_{\text{Stimulated emission}} - \underbrace{N_1 \rho_{\nu_{12}} B_{12}}_{\text{Absorption}}$$

$$\rho_{\nu_{12}} = p_{12} (h\nu_{12})$$ each photon has this energy

$$p_{12} = \frac{\rho_{\nu_{12}}}{h\nu_{12}}$$

$$\frac{1}{h\nu_{12}} \frac{d\rho_{\nu_{12}}}{dt} = \rho_{\nu_{12}} (N_2 B_{21} - N_1 B_{12})$$

If $B_{12} = B_{21} = B$

$$\frac{1}{h\nu_{12}} \frac{d\rho_{\nu_{12}}}{dt} = \rho_{\nu_{12}} B (N_2 - N_1)$$

$$\frac{d\rho_{\nu_{12}}}{dt} = \rho_{\nu_{12}} B . h\nu_{12} (N_2 - N_1)$$

$$\int \frac{\frac{d\rho_{\nu_{12}}}{dt}}{\rho_{\nu_{12}}} = \int B . h\nu_{12} (N_2 - N_1)$$

$$\int \frac{d\rho_{\nu_{12}}}{\rho_{\nu_{12}}} = \int B . h\nu_{12} . (N_2 - N_1) dt$$

$$\ln \rho_{\nu_{12}} - \ln \rho_{\nu_0} = B . h\nu_{12} (N_2 - N_1) . t$$

$$\ln \left(\frac{\rho_{\nu_{12}}}{\rho_{\nu_0}}\right) = B . h\nu_{12} (N_2 - N_1) t$$

$$e \qquad e$$

$$\boxed{\rho_{\nu_{12}} = \rho_{\nu_0} \exp [B h\nu_{12} (N_2 - N_1) t]}$$

$$\rho_{\nu_{12}} = \rho_{\nu_0} \exp\left[B \cdot h\nu_{12} \cdot (N_2 - N_1)\, t \right]$$

as $\underline{N_2 > N_1}$ $\rho_{\nu_{12}}$ increases.

if this photon field (wave) moves with the speed of light (c) then it travels $\underline{s = ct}$ in time t. $t = \frac{s}{c}$

$$\boxed{\rho_\nu = \rho_{\nu_0} \exp\left[\frac{B h\nu}{c} \cdot (N_2 - N_1)\, s \right]}$$

Since the intensity of the field is proportional to ρ_ν

$$I \propto \rho_\nu$$

$$I = I_0 \cdot \exp(\beta \cdot x)$$

β ↓ amplification factor ⇒ GAIN COEFFICIENT

x ↓ distance (s)

$$\beta \propto \frac{h\nu}{c} \cdot B \cdot (N_2 - N_1)$$

$$\boxed{\beta \propto (N_2 - N_1)}$$

* Laser is actually an AMPLIFIER. It amplifies the photon field (LIGHT)

Light Amplification by Stimulated Emission of Radiation (LASER)

If we find a way to feedback the signal properly we can obtain an <u>oscillator</u> from an <u>amplifier</u>

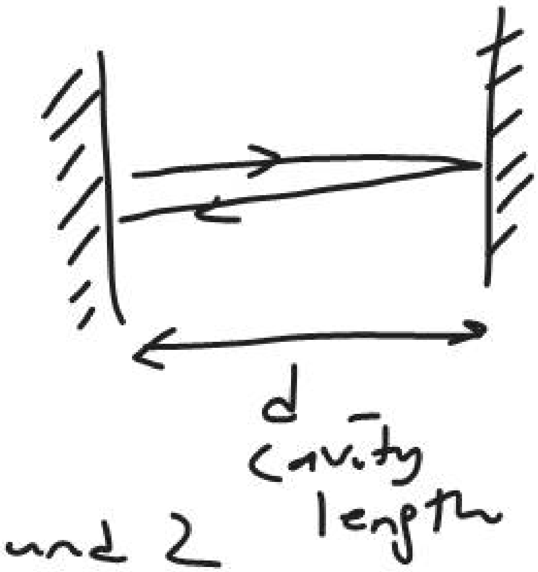
The feedback can be obtained by using the Resonator and the system continuously produces a beam of identical photons

Photons of
- same energy
- equal phase
- same direction
- ⋮

A LASER is actually the oscillator of a light of constant frequency, <u>equal phase</u>, and collimated beam of light
(coherent)

$$\frac{I_f}{I_i} = R_1 R_2 \exp\left[(\beta - \alpha)\, 2d\right] > 1$$

$$\beta \propto \frac{h\nu}{c} \cdot B\,(N_2 - N_1)$$



I_f : Final intensity of the radiation

I_i : Initial intensity of the radiation

R_1, R_2 : reflectivities of the mirrors 1 and 2

$2d$: a round trip distance in the Resonator.
(d : cavity length)

α : loss per distance (unit length) in the medium

- Narrowband in wavelength spectrum (only 1 frequency is amplified)
- The output direction is normal to the mirrors and therefore highly <u>collimated</u>.
- All photons are locked in phase, and therefore, we have a <u>coherent light</u> within the limitations of the linewidth of the transitions ($\delta\nu$)
- The light can be very <u>intense</u>

The LASER LIGHT :
- pure in wavelength and phase
- intense
- well collimated

and therefore it can be easily controlled and modulated.